TRINITY COLLEGE

Specialist Mathematics 1,2
Test 5 2017

Section 1 Calculator Free
Matrices

STUDENT'S NAME ____________________

DATE: Thursday 10 August **TIME:** 30 minutes **MARKS**: 34

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (6 marks)

Given $A=\begin{bmatrix}2 & 3\\ 1 & -4\end{bmatrix}$, $B=\begin{bmatrix}3 & 1\end{bmatrix}$, $C=\begin{bmatrix}-2\\ 7\end{bmatrix}$ and $D=\begin{bmatrix}4 & -2\\ 3 & 1\end{bmatrix}$, determine each of the following if possible. If not possible state why it cannot be done.

(a) $3A$-D $\begin{bmatrix}2 & 11\\ 0 & -13\end{bmatrix}$ [2]

(b) CB $\begin{bmatrix}-6 & -2\\ 21 & 7\end{bmatrix}$ [2]

(c) C^2 NOT POSSIBLE [2]

2×1 2×1 INCOMPATABLE DIMENSIONS

2. (10 marks)

(a) Consider the matrices $A=\begin{bmatrix}3 & 0\\ -2 & x\end{bmatrix}$, $B=\begin{bmatrix}2\\ -5\end{bmatrix}$ and $C=\begin{bmatrix}-4 & 2\end{bmatrix}$. Determine the value of x for each of the following.

(i) $A+BC=\begin{bmatrix}-5 & 4\\ 18 & -2\end{bmatrix}$ [3]

$$\begin{bmatrix}-5 & 4\\ 18 & x-10\end{bmatrix} = \begin{bmatrix}-5 & 4\\ 18 & -2\end{bmatrix}$$

$$x-10 = -2$$

$$x = 8$$

(ii) $CAB=[12]$ [3]

$$\begin{bmatrix}-4 & 2\end{bmatrix}\begin{bmatrix}3 & 0\\ -2 & x\end{bmatrix}\begin{bmatrix}2\\ -5\end{bmatrix} = [12]$$

$$\begin{bmatrix}-16 & 2x\end{bmatrix}\begin{bmatrix}2\\ -5\end{bmatrix} = [12]$$

$$[-32-10x] = [12]$$

$$-10x = 44$$

$$x = -\frac{44}{10}$$

(b) If $P=\begin{bmatrix}4 & -2\\ 1 & 0\end{bmatrix}$ and $Q=\begin{bmatrix}3 & 2\\ 6 & 4\end{bmatrix}$ determine X given that $2XP+P=Q$ [4]

$$2XP = Q-P$$

$$2XP = \begin{bmatrix}-1 & 4\\ 5 & 4\end{bmatrix}$$

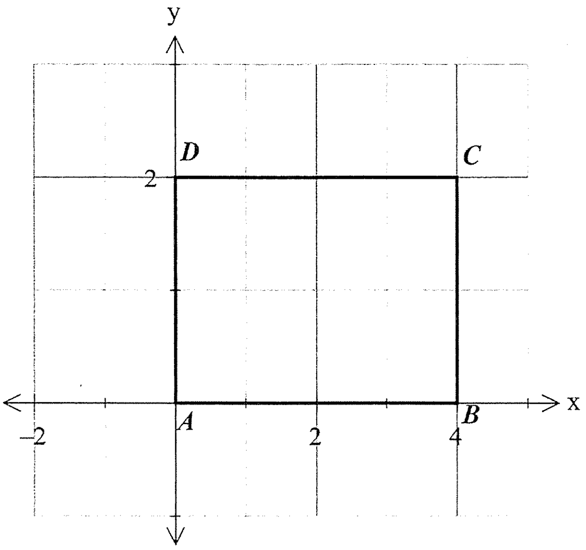
$$2X = \begin{bmatrix}-1 & 4\\ 5 & 4\end{bmatrix}P^{-1}$$

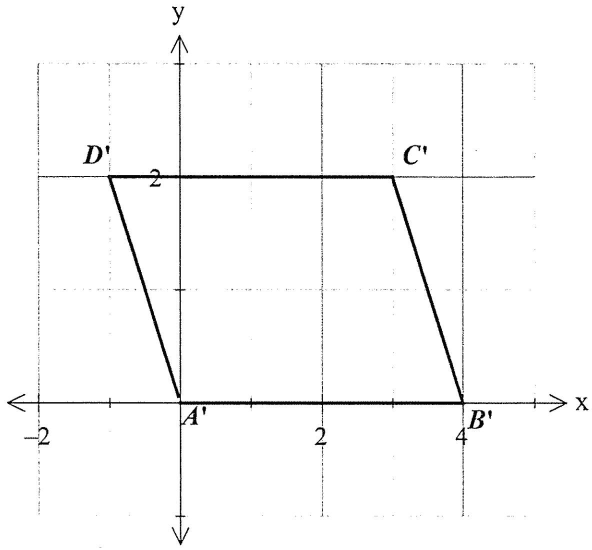
$$2X = \begin{bmatrix}-1 & 4\\ 5 & 4\end{bmatrix}\frac{1}{2}\begin{bmatrix}0 & 2\\ -1 & 4\end{bmatrix}$$

$$2X = \frac{1}{2}\begin{bmatrix}-4 & 14\\ -4 & 26\end{bmatrix}$$

$$X = \frac{1}{4}\begin{bmatrix}-4 & 14\\ -4 & 26\end{bmatrix}$$

3. (5 marks)





The rectangle was transformed into a parallelogram using a shear matrix S given by

$$S = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$$

(a) Determine the value of k [2]

$-\frac{1}{2}$

(b) If the parallelogram is transformed back to a rectangle using shear matrix T, determine T. [2]

$\begin{bmatrix} 1 & \frac{1}{2} \\ 0 & 1 \end{bmatrix}^{-1} = \frac{1}{1}\begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}$

(c) What is the relationship between S and T? [1]

$ST = I$

4. (13 marks)

(a) Let $A=\begin{bmatrix}1 & 2\\ -3 & 0\end{bmatrix}$ and $B=\begin{bmatrix}x & -2\\ y & 5\end{bmatrix}$. Determine the value of x and y such that A and B are commutative, i.e. $AB=BA$. [4]

$$\begin{bmatrix}1 & 2\\ -3 & 0\end{bmatrix}\begin{bmatrix}x & -2\\ y & 5\end{bmatrix} = \begin{bmatrix}x & -2\\ y & 5\end{bmatrix}\begin{bmatrix}1 & 2\\ -3 & 0\end{bmatrix}$$

$$\begin{bmatrix}x+2y & 8\\ -3x & 6\end{bmatrix} = \begin{bmatrix}x+6 & 2x\\ y-15 & 2y\end{bmatrix}$$

$2x = 8$ $\quad$ $2y = 6$

$x = 4$ $\quad$ $y = 3$

(b) Given that M is a 2×2 matrix such that $M^2 = M - I$, show that $M^4 = -M$. [3]

$$M^4 = M^2 M^2$$
$$= (M-I)(M-I)$$
$$= M^2 - 2MI + I^2$$
$$= (M-I) - 2M + I$$
$$= -M$$

(c) Determine the image of the line $y = -2x+1$ after being transformed by $\begin{bmatrix}0 & 2\\ -1 & 1\end{bmatrix}$. [3]

$$\begin{bmatrix}0 & 2\\ -1 & 1\end{bmatrix}\begin{bmatrix}k\\ -2k+1\end{bmatrix}$$

$$= \begin{bmatrix}-4k+2\\ -k-2k+1\end{bmatrix}$$

$\therefore\ x = -4k+2$

$4k = 2-x$

$k = \frac{2-x}{4}$

$y = -3k+1$

$= -3\left(\frac{2-x}{4}\right)+1$

$= -\frac{6}{4} + \frac{3x}{4} + 1$

$y = \frac{3x}{4} - \frac{1}{2}$

(d) Solve for X given $\underset{1\times2}{X}\underset{2\times3}{\begin{bmatrix}4 & 0 & 4\\ 0 & -1 & 1\end{bmatrix}} = 2\underset{1\times3}{\begin{bmatrix}5 & 5 & 0\end{bmatrix}}$ [3]

$$\begin{bmatrix}a & b\end{bmatrix}\begin{bmatrix}4 & 0 & 4\\ 0 & -1 & 1\end{bmatrix} = \begin{bmatrix}10 & 10 & 0\end{bmatrix}$$

$$\begin{bmatrix}4a & -b & 4a+b\end{bmatrix} = \begin{bmatrix}10 & 10 & 0\end{bmatrix}$$

$4a = 10$ $\quad$ $-b = 10$

$a = \frac{5}{2}$ $\quad$ $b = -10$

TRINITY COLLEGE

Specialist Mathematics 1,2
Test 5 2017

Section 2 Calculator Assumed
Matrices

STUDENT'S NAME SOLUTIONS

DATE: Thursday 10 August **TIME:** 25 minutes **MARKS:** 25

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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5. (5 marks)

If $A = \begin{bmatrix} 2 & 0 & 1 \\ -3 & 1 & 2 \\ 8 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & -1 \\ 19 & -6 & -7 \\ -8 & 0 & 2 \end{bmatrix}$

(a) calculate AB [2]

$$\begin{bmatrix} -6 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & -6 \end{bmatrix} = -6I$$

(b) solve the following set of simultaneous equations using matrix methods and the result from (a). [3]

$$x - z = -1$$
$$19x - 6y - 7z = -7$$
$$-8x + 2z = 8$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 19 & -6 & -7 \\ -8 & 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ -7 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{6} A^{-1}\begin{bmatrix} -1 \\ -7 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ 0 \end{bmatrix}$$

6. (10 marks)

(a) $O(0,0)$, $A(6,1)$ and $B(5,3)$ are the vertices of a triangle. Triangle OAB is transformed to triangle $OA'B'$ where A' is $(6,19)$ and B' is $(5,18)$, by transformation T_1. Determine and describe matrix T_1. [3]

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 6 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 19 & 18 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$$

SHEAR // y-AXIS SCALE FACTOR 3

(b) Triangle $OA'B'$ is transformed by matrix $T_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ to triangle $OA''B''$. Determine points A'' and B''. [2]

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 6 & 5 \\ 19 & 18 \end{bmatrix} = \begin{bmatrix} 19 & 18 \\ 6 & 5 \end{bmatrix}$$

(c) Determine a single matrix that will transform triangle OAB directly to triangle $OA''B''$. [2]

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 1 & 0 \end{bmatrix}$$

(d) Triangle $OA''B''$ is now transformed by matrix T_3 to triangle $OA'''B'''$ so that it is six times the area of the original triangle OAB. Determine three possible matrices for T_3. [3]

NO CHANGE IN AREA FROM OAB TO $OA''B''$

$\therefore$ ANY MATRIX T_3 WITH $|\text{DET}| = 6$

eg $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$, $\begin{bmatrix} 6 & 0 \\ 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 \\ 0 & 6 \end{bmatrix}$,

$\begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix}$ ....

7. (10 marks)

Matrix A = $\begin{bmatrix} 6 & 2 \\ 3 & 7 \end{bmatrix}$

(a) Show $A^2 - 13A + 36I = 0$ where I is the identity matrix and 0 is the zero matrix. [3]

$$\begin{bmatrix} 6 & 2 \\ 3 & 7 \end{bmatrix}^2 - 13\begin{bmatrix} 6 & 2 \\ 3 & 7 \end{bmatrix} + 36\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

(b) Use (a) to show $A = (A - 6I)^2$. [2]

$$A^2 - 13A + 36I = 0$$
$$A^2 - 12A + 36I = A$$
$$(A - 6I)^2 = A$$

(c) Use the result from (b) to determine a square root of A. [3]

$$A = (A - 6I)^2$$
$$A^{\frac{1}{2}} = (A - 6I)$$
$$= \begin{bmatrix} 6 & 2 \\ 3 & 7 \end{bmatrix} - \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix}$$

(d) Determine a second square root of *A*. [2]

$$\begin{bmatrix} 6 & 2 \\ 3 & 7 \end{bmatrix}^{\frac{1}{2}} = \begin{bmatrix} 2.4 & 0.4 \\ 0.6 & 2.6 \end{bmatrix}$$

or $A = (6I - A)^2$

$\sqrt{A} = 6I - A = \begin{bmatrix} 0 & -2 \\ -3 & -1 \end{bmatrix}$